Exercise Sheet 3 : Amplitude Modulation

Exercise 1

An AM transmitter broadcast the following signal:

$$s_{AM}(t) = 100\cos(2\pi \cdot 3.55 \cdot 10^6 t) + 43.5\cos(2\pi \cdot 3.545 \cdot 10^6 t) + 43.5\cos(2\pi \cdot 3.555 \cdot 10^6 t)$$

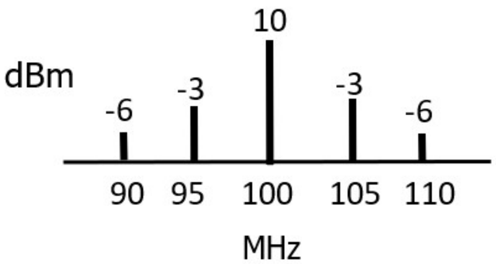
1. Determine the Upper Side-Band frequency and the modulating frequency.
2. Calculate the modulation index and the bandwidth of the modulated signal.
3. Plot the waveform of this signal and its spectrum.
4. Calculate the power of the carrier and each sideband if the transmitted power is 38W.

Exercise 2

On an oscilloscope, we observe the variations of an AM signal. A maximum voltage of 4.5V and a minimum voltage of 0.5V are read, with an HF frequency of 100kHz and a LF frequency of 10kHz. Deduce the mathematical expression of the signal and plot its spectrum.

Exercise 3

The spectrum of an AM signal is displayed on a spectrum analyzer :

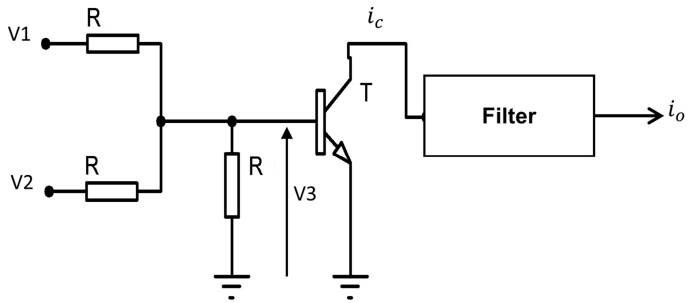


The input resistance of the spectrum analyzer is 50Ω.

1. Determine the carrier frequency and the useful signal frequency.
2. Determine the bandwidth of the useful signal.
3. Calculate the useful power and the modulation efficiency.
4. Write the mathematical expression of this signal.
5. Calculate the modulation index.

Exercise 4

Consider the following circuit:

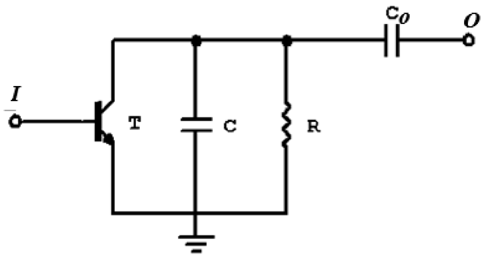


1. Determine the expression for the voltage V_3 as a function of V_1 and V_2.
2. Using a Taylor series expansion of the exponential function: $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!}$, find the expression for the current i_c as a function of v_{be}.
3. Deduce the expression of i_c as a function of V_1 and V_2.
4. Given: $V_1(t) = \cos(\omega_1 t)$ and $V_2(t) = A\cos(\omega_2 t)$ with $\omega_1 \gg \omega_2$. Plot the spectrum of i_c.
5. On the same spectrum, plot the filter response suitable for amplitude modulation with this circuit. Provide its center frequency and bandwidth.
6. Find the expression for i_o, the current at the filter output, and deduce the modulation index.
7. Calculate the average power of the AM signal represented by i_o when it passes through a 50Ω resistance.

Exercise 5

At the input of the circuit below, the voltage applied is:

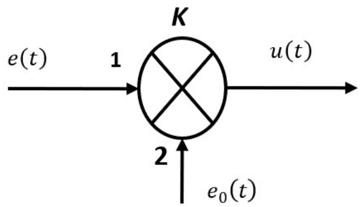
$$v_i(t) = A_c \cos(\omega_c t) + A_m \cos(\omega_c t + \omega_m t)$$



1. Calculate $v_o(t)$.
2. Deduce the function of this circuit and the characteristics of the output filter.

Exercise 6

To demodulate a signal $e(t)$, it is injected into a mixer with a gain K, whose second input comes from a local oscillator with the same frequency ω_0 as the carrier $e_0(t) = E_0 \sin(\omega_0 t)$.

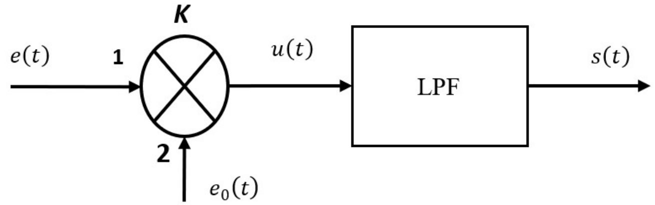


The output of the mixer is given by : $u(t) = K \cdot e(t) \cdot e_0(t)$

The modulated signal is: $e(t) = E(1 + m\cos(\Omega t))\sin(\omega_0 t)$, where F is the frequency of the modulating signal $F = \frac{\Omega}{2\pi}$ and $f_0 = \frac{\omega_0}{2\pi}$ is the carrier frequency.

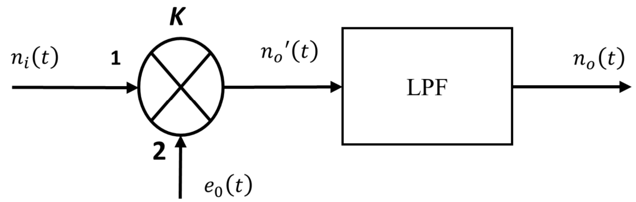
1. Write the expression for $u(t)$ and plot its frequency spectrum.

2. If $f_0 = 1$ MHz and $F = 1$ kHz, the mixer is followed by a low-pass filter with a cutoff frequency of 1500 Hz and DC component suppression. What is the expression for its output voltage $s(t)$?



3. Calculate the average power S of the signal $s(t)$.

To measure the noise at the output of the demodulator, the following circuit is used in the absence of the modulated signal:



The low-pass filter at the output of the mixer is the same as in question 2. Assume that the noise $n_i(t)$ is sinusoidal with a frequency f_n such that $f_0 - F < f_n < f_0 + F$. The expression of the noise $n_i(t)$ is: $n_i(t) = A_n \sin(\omega_n t)$, with $f_n = \frac{\omega_n}{2\pi}$.

4. Write the expression of $n'_o(t)$ at the output of the mixer. What is its spectrum?
5. Deduce the expression of $n_o(t)$ at the output of the low-pass filter.
6. Calculate the average noise power at the output N.
7. From these calculations, deduce the signal-to-noise ratio at the output $(S/N)_o$ in dB if $E = 10$ V, $A_n = 0.8$ V, and $m = 30\%$.