Exercise Sheet 4 : Frequency Modulation

Exercise 1

Given the expression of a frequency-modulated signal:

$$s(t) = 10\cos\left(6283200t - 5\cos(3141t)\right)$$

Determine:

1. The expression of its instantaneous frequency.
2. The carrier frequency and the modulating signal frequency.
3. The frequency deviation and the peak deviation.
4. The modulation index.
5. The spectrum of the modulated signal and its bandwidth.

Exercise 2

Given a carrier signal: $s_c(t) = 5\cos(2\pi 450 \times 10^3 t)$

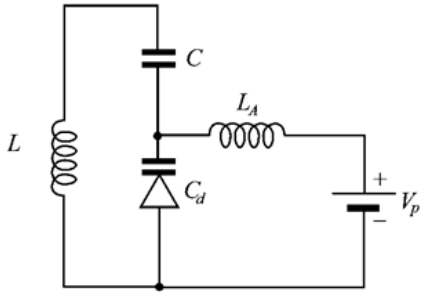
This carrier is frequency-modulated by: $s_m(t) = A\cos(6\pi 10^3 t)$ with a modulation index $m = 1$.

1. Find the amplitude of the modulating signal if it is amplified 1000 times before modulation.
2. Calculate the instantaneous frequency of the FM signal $s_{\text{FM}}(t)$, then deduce its expression.
3. Calculate the maximum frequency, minimum frequency, peak frequency deviation, and bandwidth of $s_{\text{FM}}(t)$.
4. Sketch the spectrum of the FM signal.
5. The response of a tuned circuit has a nearly linear slope, with a gain of 0.8 at 500 kHz and 0.6 at 400 kHz. Determine the expression for this slope.
6. We apply $s_{FM}(t)$ at the input of this tuned circuit. Determine the expression of the output signal. How can we recover $s_m(t)$?

Exercise 3

Consider an oscillator based on a varicap diode with an operating frequency of :

$$f_0 = \frac{1}{2\pi\sqrt{L(C \parallel C_d)}}$$



The capacitance of the varicap diode is given by :

$$C_d = \frac{k}{\sqrt{V_0 + V_p}}$$

where k and V_0 are constants, and V_p is the bias voltage of the diode. We give the following values: $L = 0.32\ \mu H$, $\quad C = 10\,000\ pF$, $\quad V_0 = 0.36\ V$, $\quad k = 9 \times 10^{-10}$

1. Express the oscillation frequency f_0 of the circuit and show that it can be written in the form:
$$f_0(V_p) = A\sqrt{B + \sqrt{D + V_p}}$$
Find the values of A, B, and D.
2. Determine the value of V_p to achieve an oscillation frequency $f_0 = 15$ MHz.
3. The voltage V_p is the sum of a DC voltage $V = 7$ V and a sinusoidal voltage $v(t)$ with an amplitude of $V_m = 0.2$ V and a frequency of $f_m = 20$ kHz.
   (a) Show that the oscillator is frequency-modulated. Determine the carrier frequency.
   (b) Calculate the modulation index of the resulting FM signal and the bandwidth necessary to transmit this signal.